L3(3.3)-Quadratic Relations in Factored Form

Key Concepts:

- factored form of quadratic relation
- direction of opening from 'a'
- solving for zeroes

- using symmetry to find:
  - x-coordinate of vertex
  - axis of symmetry

- using substitution to find:
  - y-coordinate of vertex
  - y-intercept

Is $y = 2(x+1)(x-5)$ a quadratic relation?

Examine 1st and 2nd differences:

x	y	1st	
-2	14		
-1	0	$0-14=-14$	
0	-10	$-10-0=-10$	$-10-(-14)=4$
1	-16	$-16-(-10)=-6$	$-6-(-10)=4$
2	-18	$-18-(-16)=-2$	$-2-(-6)=4$

Quadratic if 2nd differences are constant

Quadratic Relations in Factored Form

The equation of a quadratic relation may be written in several forms:

1. standard form: $y = ax^2 + bx + c$
2. factored form: $\boxed{y = a(x - s)(x - t)}$
3. vertex form: $y = a(x - h)^2 + k$

The factored form, $y = a(x - s)(x - t)$, is most useful for finding the zeroes, which are $x = s$ and $x = t$.

Consider the following...

Give two numbers that have a product of zero:

★ $0(10) = 0$ $(0)(0) = 0$

$10(0) = 0$ $(5+5) = 0$

What do you notice? $(\text{any value}) \times 0 = 0$

★ only way to get a product of zero is x 0

Solve:

(a) $3x = 0$

$\frac{3x}{3} = \frac{0}{3}$

$x = 0$

(b) $57y = 0$

$\frac{57y}{57} = \frac{0}{57}$

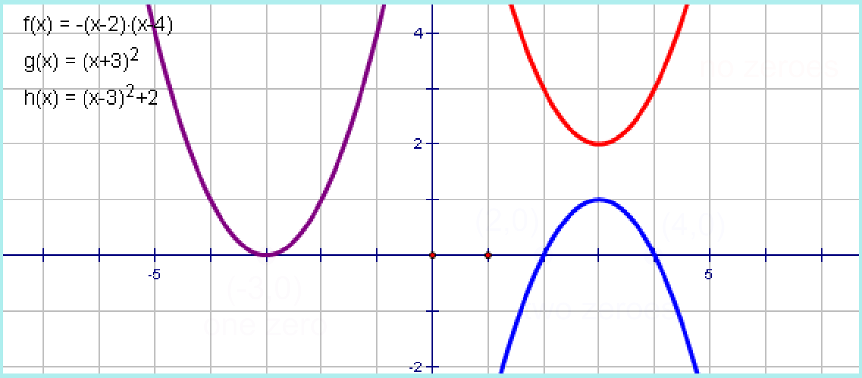
$y = 0$

(c) $3xy = 0$

$\frac{3xy}{3} = \frac{0}{3}$

$xy = 0$

Depending upon the location of the vertex, and whether the parabola opens up or down, it may have 0, 1, or 2 distinct (unique) zeroes.



Zeroes occur where the y-coordinate of the parabola is equal to zero.

To find the zeroes algebraically, we **set y = 0** and solve for the x-values that make the equation true.

Ex.1 Determine the zero(es) of each

(a) $y = x(x - 10)$

$0 = x(x-10)$

$x=0$ or $x-10=0$

$x=10$

Recall:

Zero multiplied by anything is zero.

If (a)(b) = 0 then
a = 0 or b = 0 (or both are zero).

(b) $y = -2(x - 5)(3x - 1)$

$0 = -2(x-5)(3x-1)$

$x-5=0$ or $3x-1=0$

$\boxed{x=5}$

$3x=1$

$\frac{3x}{3} = \frac{1}{3}$

$\boxed{x=\frac{1}{3}}$

(c) $y = 2(x - 2)^2$

$0 = 2(x-2)(x-2)$

$x-2=0$ or $x-2=0$

$\boxed{x=2}$ $\boxed{x=2}$

One zero

The zeroes and symmetry can be used to find the vertex (h, k).

For the x-coordinate (h), find the midpoint of the zeroes:

☆ $$MP = \frac{x_1 + x_2}{2} = \frac{s + t}{2}$$

For the y-coordinate (k), substitute the midpoint into the equation and solve for y:

☆ $$y = a(x - s)(x - t)$$

$$y = a(MP - s)(MP - t)$$

Ex.2 Determine the vertex:

(a) y = -2(x - 2)(x - 8)

$0 = -2(x-2)(x-8)$

$x-2=0$ or $x-8=0$

$\boxed{x=2}$ $\boxed{x=8}$

$AoS = \frac{2+8}{2}$

$x = 5$

Sub x=5 into equation

$y = -2(5-2)(5-8)$

$= -2(3)(-3)$

$= +18$

∴ the vertex is (5, 18)

Ex.3 A parabola has zeroes at -3 and 2, and a y-intercept of 18. Determine the equation.

= (0, 18)

$y = a(x-s)(x-t)$

$y = a(x+3)(x-2)$

Sub in (0, 18)

$18 = a(0+3)(0-2)$

$18 = a(3)(-2)$

$\frac{18}{-6} = \frac{-6a}{-6}$

$a = -3$

∴ the equation is

$y = -3(x+3)(x-2)$

Assigned Work: p. 155-157 # 2, 3, 4ace, 5, 6ace, 7, 10